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# On different actions for the vacuum of bosonic string field theory

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Abstract: We study a family of kinetic operators in string field theory describing the theory around the closed string vacuum. Those operators are based on the analytical classical solutions of Takahashi and Tanimoto and are analogous to the pure ghost action usually referred to as "vacuum string field theory," but are much more general, and less singular than the pure ghost operator. The closed string vacuum is related to the D-brane vacuum by large, singular, gauge transformations or field redefinition, and all those different representations are related to each other by small gauge transformations. We try to clarify the nature of this singular gauge transformation. We also show that by choosing the Siegel gauge one recovers the propagator proposed in [10] that generates closed string surfaces.

Keywords: Bosonic Strings, Tachyon Condensation, String Field Theory.

JHEP08(2003)017

## Contents



## 1. Introduction

The usefulness of string field theory [1] as more than just a rewriting of first quantized open strings was demonstrated in recent years. There is little doubt now that string field theory includes classical solutions, or vacua, with any number of D-branes. The theory is defined around the D-brane vacuum and a lot of work has gone into finding the description around the lower vacuum, without the D-brane.

A classical solution describing this vacuum was found in successive numerical approximations using level truncation in the Siegel gauge based on [2] (for the most detailed calculation see [3] and references therein). Another approach to the problem [4] was to guess the form of the action around the new vacuum. Since the action is cubic, by expanding it around the classical solution $\Phi_0$ one finds the same form of the action but instead of the BRST charge $Q_B$ the kinetic operator is given by

$$\mathcal{Q}\Psi = Q_B\Psi + \Phi_0 \star \Psi - (-1)^{\Psi}\Psi \star \Phi_0 \,. \tag{1.1}$$

One then took the simplifying assumption that $\mathcal{Q}$ is pure ghost, and found [5, 6] that it should be the midpoint insertion of $c$, that is $c(\pi/2)$.

Using this action one could construct the classical solutions describing D-branes and calculate their actions [7].

The situation is still not fully satisfactory. For one, it is of interest to find the analytical form of the solution starting from the open string action. In principle, if one identifies all the open string states living on the sliver one would have the full field redefinition between the two forms of the the action, and the analytical classical solution, but that seems very hard and tedious. The second objection is that the pure ghost kinetic term is very singular, so one might hope to find nicer expressions for the closed string vacuum.



JHEP08(2003)017

This was in fact done in a very nice paper of Takahashi and Tanimoto [8]. They construct classical solutions of bosonic open string field theory by acting with certain combinations of the BRST current and the ghost on the identity state. They were able to show that those solutions are gauge transformations of the usual vacuum, but sometimes those gauge transformations are singular, and lead to non-trivial solutions.

They also used those solutions to find the kinetic operator around the new vacuum, as in (1.1). Again those are sometimes equivalent to the regular kinetic operator $Q_B$, but when the gauge transformation is singular, they are not. In particular, in one case it was shown in [9] that this new kinetic operator has trivial cohomology at the right ghost number. This is the hallmark of the kinetic operator around the closed string vacuum, since we should not find any on-shell open strings in the spectrum.

In this paper we study those kinetic operators.

The classical solutions they found are based on the identity state, and are rather hard to control. For example the value of the classical action, which should be minus the tension of the D-25 brane, has not yet been calculated. But the kinetic operators are smooth and in many ways nicer than the pure ghost $c(\pi/2)$.

The next section is devoted to the basic properties of those kinetic operators. We present them as constructed in [8], as a convolution of the BRST current with some function $f$ (plus a pure ghost term). There is such an operator $\mathcal{Q}_f$ for any function $f$ on the interval $[0, \pi]$ which is equal to one at the midpoint and symmetric around it. We also review how these operators are related to the usual $Q_B$ by a field redefinition.

We then study some properties of those field redefinitions and under what conditions they are singular. The answer turn out to be very simple, that the function $f$ have a zero or a pole.

One example we point out is that by a singular limit on the function $f$, taking it to be zero everywhere except the midpoint, we can get the pure ghost operator of [5, 6].

In section 3 we study the gauge fixed action. By choosing the Siegel gauge one finds the gauge fixed kinetic operator which includes the convolution of the energy momentum tensor with the function $f$. This in fact fits beautifully with the ideas in [10], where a similar type of gauge-fixed kinetic operator was derived from totally different considerations. There it was argued that the kinetic operator will not be gauge equivalent to the usual one if $f$ has zeros on the boundaries of the world-sheet. And instead of open surfaces, the Feynman rules derived from this action will generate closed surfaces.

By starting with the exact gauge invariant form of the action we will fix some extra terms in the propagator, related to the ghosts, which were not determined in [10].

Another point, related to closed string vertices, is clarified in section 4. The open-closed vertices of Shapiro and Thorn are gauge invariant only when they commute with the kinetic operator. In the D-brane vacuum the commutator with $Q_B$ leads to the mass-shell condition for the closed string operators. The singular pure ghost operator does not give this constraint on the closed string spectrum, but we show here that all the $\mathcal{Q}_f$ do indeed satisfy this condition and pick out only on-shell closed string vertices.

In section 5 we go back to the singular field redefinitions and try to elucidate how the zeros of $f$ lead to the non-trivial vacua. The field redefinition that puts the action back in the original form, with the BRST charge involves an operator built out of the ghost current and the logarithm of $f$. When multiplying fields with the star product those operators cancel each other for regular $f$. But for singular functions the operators are defined on different branches on the logarithm, so they cannot cancel each other completely.

We learn two things from this calculation. First, it proves that only the singularity structure of $f$ matters. Small gauge transformations can change the value of $f$, but adding and removing zeros are large gauge transformations. We conjecture that the number of zeros and their order is related to the number of D-branes, or the rank of the projector. Second, it allows us the write the action around the closed string vacuum in a new way. The new action has the usual kinetic operator $Q_B$, but a modified gluing rule in the star product and integration.

The last section 6 is devoted to some discussion and more speculations.

## 2. Kinetic operator around the vacuum

### 2.1 Review

In [8] Takahashi and Tanimoto constructed a family of classical solutions to Witten's cubic string field theory. Then they proceeded to formulate the action for small fluctuations around their classical solutions. Since the construction is rather formal and technical, I will not repeat it here, but refer the reader the the original paper. The outcome of the calculation is an action like Witten's

$$S = \int \Phi \star \mathcal{Q}_f \Phi + \frac{2}{3} \Phi \star \Phi \star \Phi \,. \tag{2.1}$$

Here $\Phi$ is the string field shifted by the classical solution $\Phi_0$, and the integration and star product are the regular gluing rules for string fields [1]. The only new ingredient is the kinetic operator $\mathcal{Q}_f$ defined in terms of the BRST current $j_B$ and the ghost $c$ as

$$\begin{aligned} \mathcal{Q}_f &= \frac{1}{2\pi} \int_0^{\pi} d\sigma \left[ (j_B(\sigma) + \bar{j}_B(\sigma)) f(\sigma) - (c(\sigma) + \bar{c}(\sigma)) \frac{f'(\sigma)^2}{f(\sigma)} \right] \\ &= \frac{1}{2\pi i} \oint dw \left[ j_B(w) f(w) - c(w) \frac{f'(w)^2}{f(w)} \right] . \end{aligned} \tag{2.2}$$

In the first line we wrote the integral over the coordinate $\sigma$ on the strip, and in the second line we represented the string field as half the unit disc in the upper half plane. The coordinates are related by $w = \exp i\sigma$, and we included the antiholomorphic piece by an image in the lower half plane.

The particular choice $f = 1$ gives the usual kinetic operator $Q_B = 1/(2\pi i) \oint dw j_B(w)$.

The fact that $\mathcal{Q}_f$ is nilpotent and a derivation of the star algebra are direct consequences of its construction as a shift of the regular kinetic operator (1.1). But it is also not too hard to verify it directly. Using the OPEs

$$\begin{aligned} j_B(z)j_B(w) &\sim \frac{-4}{(z-w)^3}c\partial c(w) - \frac{2}{(z-w)^2}c\partial^2 c(w) = -\partial_w\left(\frac{2}{(z-w)^2}c\partial c(w)\right), \\ j_B(z)c(w) &\sim \frac{1}{z-w}c\partial c(w)\,, \end{aligned} \tag{2.3}$$

one immediately sees that the commutator of the term coming from the two currents cancels the cross term.

The other important property is that $\mathcal{Q}_f$ be a derivation of the star algebra, that is $\mathcal{Q}_f(A\star B) = \mathcal{Q}_f A\star B + (-1)^A A\star \mathcal{Q}_f B$. This can be expressed in terms of the three-string vertex $\langle V_3|$ as

$$\langle V_3|\left(\mathcal{Q}_f^{(1)} + \mathcal{Q}_f^{(2)} + \mathcal{Q}_f^{(3)}\right) = 0\,, \tag{2.4}$$

where the superscript on $\mathcal{Q}_f$ indicates which of the three strings it acts on. This equation is a consequences of the conservation laws derived in [11] for the $c$ ghost, and the analogous equalities for the BRST current. When considering the pure ghost operator [4] this imposed the constraint that only the combinations $\mathcal{C}_n = c_n + (-1)^n c_{-n}$ of the modes of the ghost appear. A similar constraint exists for the modes of the BRST current. Those imply that the function $f(w)$ has to satisfy $f(-1/w) = f(w)$.

Using an identical argument one can show that $\mathcal{Q}_f$ annihilates the identity state. This implies that the new action has the gauge invariance (at the linear level) of a shift by $\mathcal{Q}_f\Lambda$ for any $\Lambda$.

Another condition on $f$ that comes out of their construction is that $f(i) = f(-i) = 1$. We have not found an independent explanation of it.

### 2.2 Singular field redefinitions

As is pointed out in [8], one can relate $\mathcal{Q}_f$ to the usual $Q_B$ by the similarity transformation

$$\mathcal{Q}_f = e^{q(h)}Q_B e^{-q(h)}\,, \tag{2.5}$$

where $q(h)$ is an operator constructed out of the ghost current $j_{gh} = cb$ convoluted with $h = \log f$ as

$$q(h) = \frac{1}{2\pi i}\oint dw j_{gh}(w)h(w)\,. \tag{2.6}$$

The proof follows from repeated use of the commutators $[q(h), j_B(w)] = h(w)j_B(w) + 2\partial(c(w)\partial f(w))$ and $[q(h), c(w)] = h(w)c(w)$.

In order to define $e^{q(h)}$ properly in the quantum theory they wrote it in normal ordered form. First they separate $q(h)$ into the sum of the zero mode and positive and negative modes as $q(h) = q_0(h) + q^{(+)}(h) + q^{(-)}(h)$, then one can define

$$e^{q(h)} = e^{\frac{1}{2}\left[q^{(+)}(h), q^{(-)}(h)\right]}e^{q_0(h)}e^{q^{(-)}(h)}e^{q^{(+)}(h)}\,. \tag{2.7}$$

They noted that in some cases the normal ordering constant from the commutator of the positive and negative frequencies diverges. If we expand $h$ and $j_{gh}$ as

$$h(w) = \sum h_n w^{-n}\,, \qquad j_{gh}(w) = \sum q_n w^{-n-1}\,. \tag{2.8}$$



JHEP08(2003)017

then using $[q_n, q_m] = m\delta_{m+n}$ we see that the commutator is

$$\left[q^{(+)}(h), q^{(-)}(h)\right] = \sum_{n=1}^{\infty} n h_{-n} h_n \,. \tag{2.9}$$

If the function $f$ vanishes on the unit circle, the function $h$ will have a logarithmic singularity, and the Laurent coefficients will behave as

$$h_{-n} \sim h_n \sim \frac{1}{n} \,. \tag{2.10}$$

This behavior will lead to a divergence in the normal ordering constant (2.9), and $e^{q(h)}$ will be ill defined.

We therefore conclude that if $f$ has no singularities on the unit circle the action with $\mathcal{Q}_f$ is a regular field redefinition of the usual action, so it describes the open strings on the D-25 brane. If $f$ has singular points this action could describe a different vacuum.

in particular for the function

$$f = -\frac{1}{4}\left(w - \frac{1}{w}\right)^2 = \sin^2\sigma \,, \tag{2.11}$$

it was shown in [9] that $\mathcal{Q}_f$ has trivial cohomology at ghost number one. Therefore the action with this kinetic term is an appropriate candidate to describe the closed string vacuum. Any other function with the same zeros will yield the same results, since they will be related to each other by a regular field redefinition.

It is not hard to see the source of the problem. The action of $e^{q(h)}$ on the ghost $c$ is

$$e^{q(h)} c(w) e^{-q(h)} = f(w) c(w) \,, \tag{2.12}$$

so if the function $f$ has zeros or poles on the unit circle it is a singular operator (acting on the anti-ghost $b$ will multiply it by $1/f$). We believe that the source of the singularity is the attempt to write an operator with a non-trivial kernel as an exponent. In section 5 we try to make sense of this operator by deforming the contours used to define $q(h)$.

It is not surprising that a singular transformation changes the cohomology. If we write $Q_B = e^{-q(h)} \mathcal{Q}_f e^{q(h)}$, and assume that $\mathcal{Q}_f$ has trivial cohomology. The new physical states should come from the kernel of the transformation $e^{q(h)}$.

### 2.3 Pure ghost $\mathcal{Q}$

One may wonder about the pure ghost kinetic term of [5, 6]. How is this related to those operators. In fact it is a limit of such operators.

Consider a function $f$ that is close to zero along the entire unit circle, except for the midpoint ($i$ and $-i$). For example

$$f = \frac{\left(w^2 - 1\right)^2}{\left(w^2 - 1\right)^2 - a^2\left(w^2 + 1\right)^2} \,, \tag{2.13}$$

with very large $a$. The integral of $f$ around the circle is very small (proportional to $1/a$), so there is no contribution to $\mathcal{Q}_f$ from the BRST current $\oint dw j_B f$, but the derivative of $f$ diverges near the midpoint, so there is an infinite contribution from

$$\mathcal{Q}_f \sim \oint dw \frac{(f')^2}{f} c \sim a \left( c\left(\frac{\pi}{2} - \frac{1}{a}\right) + c\left(\frac{\pi}{2} + \frac{1}{a}\right)\right). \tag{2.14}$$

In the limit this can be regarded as the pure ghost midpoint operator.

## 3. Gauge fixed kinetic operator

Given this action one may try to check different properties of it. One possible check is to derive the Feynman rules, as was done for the pure ghost operator in [6]. In particular we will compare the results with [10], where the general form of the gauge fixed action, and of the Feynman rules were proposed.

The Siegel gauge condition on a string field is $b_0\Phi = 0$, where $b_0$ is the zero mode of the anti-ghost. This means that any field satisfying the gauge condition can be written as $b_0\Psi$ for some $\Psi$. Therefore the quadratic part of the action $\langle\Psi|b_0\mathcal{Q}_f b_0|\Psi\rangle$ picks out the term in $\mathcal{Q}_f$ proportional to $c_0$, the ghost zero mode. To calculate it we use the OPE

$$j_B(z)b(w) = \frac{3}{(z-w)^3} + \frac{j_{gh}(w)}{(z-w)^2} + \frac{T(w)}{z-w} + \cdots . \tag{3.1}$$

So the anticommutator is

$$L_f = \{b_0, \mathcal{Q}_f\} = \frac{1}{2\pi i}\oint dw \left[ (T(w) - \partial j_{gh}(w))f(w) + \frac{f'(w)^2}{f(w)} \right], \tag{3.2}$$

and the gauge fixed kinetic term is just $c_0 L_f$.

The combination $T - \partial j_{gh}$ is a modification of the energy momentum tensor which corresponds to a change in the central charge of the $b, c$ system to $-2$. This is the same twist as applied in [6] to find the sliver solution to the ghost equation of motion with the pure ghost action. Using the bosonized ghost $\varphi$, with $c \sim \exp i\varphi$ and $b \sim \exp -i\varphi$, this amounts to changing the world-sheet action by

$$\frac{i}{2\pi}\int d^2\sigma \sqrt{\gamma}\, R^{(2)} \left(\varphi + \bar{\varphi}\right), \tag{3.3}$$

where $R^{(2)}$ is the Ricci scalar on the world-sheet.

This family of kinetic operators is very similar to those proposed in [10]. There it was argued that the inverse propagator should be the convolution of the energy-momentum tensor with an arbitrary function $f$, symmetric around the midpoint. If $f$ was regular the Feynman graphs would correspond to open surfaces, and therefore will describe the theory around the D-brane vacuum. If $f$ vanishes at the boundary of the world-sheet the propagator would correspond to a world-sheet where the boundary shrunk to a point, and should describe purely closed surfaces.[1]

[1]It is not clear what the interpretation is of a zero of $f$ away from the boundary $\sigma = 0, \pi$.



JHEP08(2003)017

More precisely, the propagator

$$b_0 \int dt \exp\left(-tL_f\right), \tag{3.4}$$

generates a curved worldsheet with metric $ds^2 = d\sigma^2 + f(\sigma)^2 dt^2$. This represents a segment of the round sphere if one chooses $f = \sin\sigma$ as in [10], or other metrics on it for different functions.

In [10] the kinetic operator was constructed from the regular energy-momentum tensor, but in fact much of the ghost structure was not investigated there closely. Starting from the exact gauge invariant $\mathcal{Q}_f$ we learn that we should use the modified energy-momentum tensor, and the extra constant from the pure ghost term. All the arguments go through, only that we should use the modified world-sheet action on the Riemann surfaces generated by the Feynman rules of [10], with the extra constant modifying the measure.

There is actually a very nice interpretation of the extra ghost piece. One can choose the function $f$ to be almost 1 everywhere, except at the boundaries, where it should vanish. The worldsheet generated by $L_f$ will have this profile, that is look like a capped cylinder. The constant term $\int (f')^2/f$ will be very large, and will favor short propagators. So the worldsheet will be built out of very narrow capped cylinders, much like in [6] (though there the cylinders were not capped). It was suggested that at the end of those cylinders there should be a vertex operator for a zero momentum dilaton.

This world-sheet is flat almost everywhere, except at the ends of the cylinders, where one should include the correction term in the action (3.3). That amounts to inserting $c\bar{c}$ ghosts at the end of the cylinder, as is appropriate for a closed string vertex operator.

In principle one could now use these Feynman rules and try to derive the closed string scattering amplitude on the sphere. We will not pursue this here.

## 4. Closed string vertices

In the vacuum of string field theory we expect to find no on-shell open string states. But the theory should not be empty, rather it should describe closed strings. As explained in [12, 6], closed strings should be the gauge invariant composite operators of the theory, and closed string scattering is the correlator of those operators. The relevant operators describing closed strings were constructed in [13]–[15] as follows. For any closed string vertex operator $V = c\bar{c}V_m$, where $V_m$ is a primary of dimension $(1,1)$ in the matter conformal field theory we define

$$O_V = \int V\left(\frac{\pi}{2}\right)\Phi\,. \tag{4.1}$$

So this is an insertion of a closed string vertex operator at the string midpoint.

The crucial point is that those operators should be gauge invariant if and only if $V$ describes a physical closed string. So these operators allow to add on-shell closed strings to the theory which includes off-shell open strings.



JHEP08(2003)017

The linear gauge transformations around the D-brane vacuum amount to a shift of $\Phi$ by a BRST exact field $Q_B\Lambda$. If $Q_B$ commutes with $V(\pi/2)$ we can write the variation as

$$\delta_\Lambda O_V = \int Q_B V\left(\frac{\pi}{2}\right)\Lambda = 0\,, \tag{4.2}$$

since the integral of a pure gauge quantity vanishes. Therefore the mass-shell condition for closed strings should be $[Q_B, V(\pi/2)] = 0$, as is indeed the case.

This can be used as a check for the kinetic operator of the closed string vacuum. It should satisfy $[\mathcal{Q}, V(\pi/2)] = 0$ if and only if $V$ is the vertex operator for an on-shell closed string. The pure ghost operator [6] commutes with all operators that do not involve the anti-ghost, and therefore does not constrain the spectrum of closed strings.

All the operators $\mathcal{Q}_f$ of [8] satisfy this condition. The function $f$ has to satisfy $f(\pi/2) = 1$ and $f'(\pi/2) = 0$. So near the midpoint $\mathcal{Q}_f \sim \int j_B$, which is the same as $Q_B$, and therefore the singular terms in the OPE of the current $j_B f$ with any vertex operator $V$ at the midpoint will not depend on $f$. We conclude that all those operators satisfy the important constraint

$$\left[\mathcal{Q}_f, V\left(\frac{\pi}{2}\right)\right] = 0 \quad \Leftrightarrow \quad V \text{ represents an on-shell closed string.} \tag{4.3}$$

As we noted above, the pure ghost operator is a limit of $\mathcal{Q}_f$ as $f$ approaches a singular function which is zero everywhere but at the midpoint. For any smooth approximation of this behavior the condition will be satisfied. But in the limit, when we ignore the $j_B$ contribution to $\mathcal{Q}$ and retain only the ghost part, this property is lost.

## 5. Analytic structure and new star product

The claim is that $\mathcal{Q}_f$ is equivalent to the standard BRST operator $Q_B$ for a regular $f$, but is different if $f$ has singularities. One argument comes from the Feynman rules derived from the gauged fixed action, where the condition that $f$ vanish on the boundary shrinks the boundaries of the world-sheet to a point. Another explanation was that the operator $e^{q(h)}$ used to relate the two operators is ill defined.

We want to examine this issue further, and try to see if we can still define a similarity operator $e^{q(h)}$ and understand how the zeros of $f$ effect the physics so profoundly.

Let us start with a regular function $f$. Then we know that $\mathcal{Q}_f = e^{q(h)}Q_B e^{-q(h)}$. Thus we can write the action as

$$\begin{aligned} S &= \int \Phi \star e^{q(h)} Q_B e^{-q(h)}\Phi + \frac{2}{3}\Phi\star\Phi\star\Phi \\ &= \langle V_2|e^{q(h)}Q_B\big|\Phi'\big\rangle_1 e^{q(h)}\big|\Phi'\big\rangle_2 + \frac{2}{3}\langle V_3|e^{q(h)}\big|\Phi'\big\rangle_1 e^{q(h)}\big|\Phi'\big\rangle_2 e^{q(h)}\big|\Phi'\big\rangle_3\,. \end{aligned} \tag{5.1}$$

In the second line we wrote the action in terms of the two and three string gluing vertices $\langle V_2|$ and $\langle V_3|$ and we defined $\Phi' = e^{-q(h)}\Phi$.

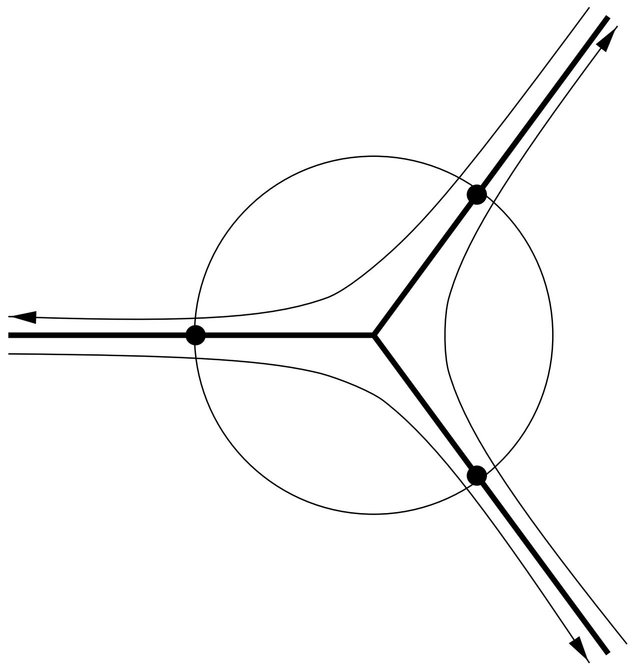

**Figure 1:** The disc is made up of three strings glued along the bold lines. The operators $q(h)$ are given by integrals along the curves with the arrows and act on each of the strings (the antiholomorphic contribution is represented by the image outside the disc). Their sum is zero, since the combination of the three contours is trivial.

Now we can follow [11] and act with the two $e^{q(h)}$ to the left on $\langle V_2|$, and with the three on $\langle V_3|$. Each $q(h)$ is an integral over a current along the boundary of the string. These boundaries are glued to each other, and apart for a numerical anomaly the sum of all the contours is a trivial closed curve.

For example, for the three string vertex we can conformally map the three discs describing the string fields to the entire plane (actually they are three half discs mapped to the unit disk, but we double the string fields to discs, and the final outcome to the entire plane to represent the antiholomorphic pieces). The midpoint of all the strings is mapped to the origin and each string fills a $2\pi/3$ wedge in the plane. The contours defining the three operators $q(h)$ are shown by the curves with arrows in figure 1. The sum of these three curves is clearly trivial.

What goes wrong if the function $f$ is singular on the unit circle? By the conformal map the unit circles are the curves along which the strings are glued, depicted by the thick lines. The function $h = \log f$ will have branch cuts going between these line and some point in the interior of the string.

The contours actually pass right through the singularities, so some regularization is needed. One option is to move the singularity away from the bold line, the other is to move the contours. The first option means that we changed $h$, or $f$, so that there is no longer a singularity along the unit circle. That will clearly lead to the trivial result. So let us try the other regularization, and move the contour away from the unit circle, slightly into the three discs. This is the picture drawn in figure 2.

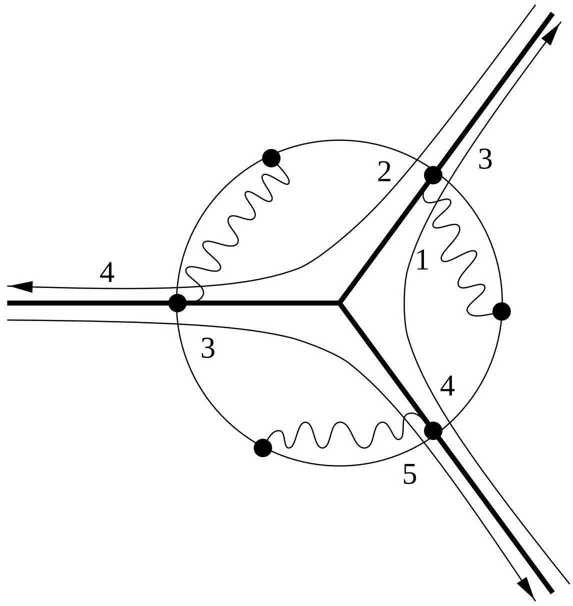


**Figure 2:** If the function $f$ has zeros along the bold lines, where the three strings are glued, there will be branch cuts for the function $h$. Those are indicated by the wiggly line in the picture. The contour defining $q(h)$ passes through the branch cuts and the three contours are not trivial anymore. In this picture we included numbers labeling a certain choice of sheets for the logarithms, assuming $f$ has a double zero.

With this regularization the contours do not cancel each other, and in fact do not close, but end on a different branch of the logarithms. Yet we can calculate the total integral, since the difference between two sheets of a log is just $2\pi i 2n$, where $2n$ is the order[2] of the zero of $f$. So the three contours cancel each other leaving some remnant integral of the ghost current times a constant. Since the curves do not close, but end on different sheets the result will depend on the chosen starting point and sheet assignment. This arbitrariness should cancel in the final result, but it is not clear how.

If we choose to start the integration near the midpoint on one of the three curves and the sheets depicted by the numbers in figure 2. we will be left with the net integral shown in figure 3. This amounts to acting on each of the three strings with the operator

$$\exp n \left[ \int_1^i dw j_{gh}(w) + \int_{-1}^{-i} dw j_{gh}(w) \right] . \tag{5.2}$$

The contours lead from the singular point (the boundary) to the midpoint.

In terms of the bosonized ghost we can write $j_{gh} = i\partial\varphi$, so we get only the boundary term

$$e^{in\varphi(i)} e^{-in\varphi(1)} e^{in\varphi(-i)} e^{-in\varphi(-1)} . \tag{5.3}$$

For $n = 1$ these are just insertions of $c\bar{c}$ at the midpoint and $b$ and $\bar{b}$ at the boundaries.

[2]An analytic function satisfying the symmetry properties must have the same number of zeros in the upper and lower half plane. If the zero is at the boundary, at $w = \pm 1$ it must be a double zero. The function $|\sin\sigma|$ chosen throughout most of [10] is not analytic, and probably not a good choice.

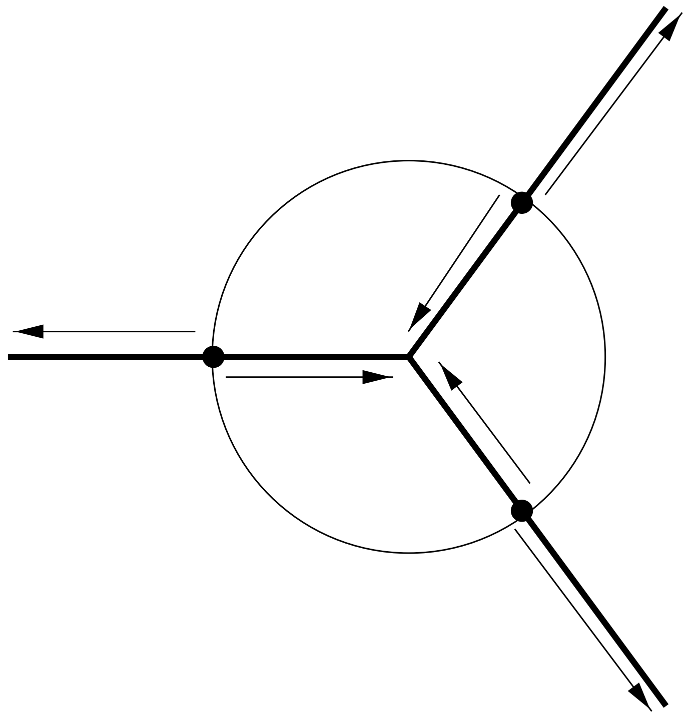

**Figure 3:** With the choice of sheets for the logarithms as in figure 2. one is left with the integral over the ghost current along the lines indicated in this picture. This is the most symmetric choice of sheets, but is not unique.

A similar thing happens to the two string vertex $\langle V_2|$. Again one gets those extra $b$ and $c$ terms at the midpoint and boundaries.

It is natural to absorb these ghost insertions in the definition of the two and three string vertices, or the star product. Thus we learn that even for a singular $f$ we can replace $\mathcal{Q}_f$ with the old $Q_B$, but the price is that we have to modify the star product (and integration). The new star product identifies the half strings as before, but also inserts $b$ and $c$ ghosts at the boundary and midpoints.

So we found a new form for the action of string field theory around the vacuum, where instead of modifying the kinetic operator, we changed the gluing rules, by adding extra ghosts. As mentioned above, there is some arbitrariness in choosing the starting point in the contour integrals of $q(h)$, which will change the type of ghost insertions. We expect this to be some symmetry that will not show up in any calculation, but the mechanism that cancels this is not clear.

One other simple prescription is to start the integral at one of the boundaries. Then all the integrals as in figure 3. and equation (5.3) will run between the boundary points, and not end at the midpoint. For the two string vertex it amounts to inserting $b\partial b$ at one boundary and $c\partial c$ at the other. In the three point vertex two boundary points get $b\partial b$ each and the third boundary point will get $c\partial c\partial^2 c\partial^3 c$. This is a less symmetric prescription, but involves fewer ghost insertions, so it might be easier to work with.

The upshot of this approach is that the dependence on the form of $f$ completely disappeared. The only remnant is the singularity structure, or the order of the zero (or more generally zeros) of $f$. So clearly all functions $f$ with a double zero at the boundary, like $f = -(w - 1/w)^2/4$ studied in [8, 9] are equivalent to each other. That is, they are related by small gauge transformations.

It is natural to conjecture that more zeros, or higher order zeros correspond to higher rank projectors, and those should give multiple D-branes. Thus we can write string field theory around any classical solution in terms of the usual $Q_B$, but with different gluing rules.

## 6. Discussion

In this paper we studied the kinetic operators $\mathcal{Q}_f$ for the vacuum of string field theory constructed by Takahashi and Tanimoto [8]. They derived them from a shift by a formal classical solution of open cubic string field theory, and in [9] it was shown that for a certain function $f$ the cohomology is trivial (at ghost number one).

These operators can serve instead of the pure ghost operator of [6]. There is a large family of operators that are analogous to each other and are less singular than the pure ghost midpoint insertion. But we also show that the pure ghost operator is a limit of $\mathcal{Q}_f$ for a singular function $f$.

We looked further into those operators. First we explained the origin of the singularity in the field redefinition that relates those $\mathcal{Q}_f$ to the BRST operator $Q_B$. Those two operators are gauge equivalent if the function $f$ is regular on the unit circle, but if $f$ has a zero the similarity transformation is singular.

Another check on these kinetic operators are the propagators derived from them. By choosing the Siegel gauge one reduces to a gauge fixed action of the form proposed in [10] from very different considerations. The fact that the function $f$ vanishes causes the boundaries of the world-sheet to shrink to points. So the Feynman graphs of the theory will be surfaces without boundaries, as is appropriate for closed strings.

The Feynman rules derived from $\mathcal{Q}_f$ give a very explicit procedure for calculating the closed string scattering amplitudes. The geometries of the world-sheets were already found in [10] and the ghost part of the world-sheet action and measure, which were not studied there, are given here. One could use these rules to calculate closed string scattering as was done for open strings in [16].

We proceeded to look at the singular field redefinition that relates $Q_B$ to $\mathcal{Q}_f$. We propose that even when $f$ is singular one can replace one operator by another, at the price of changing the gluing rules that define the star product and integration. This leads to a new form of the action with extra ghost inserted when string fields are glued.

This formalism deserves further study, as we plan to do, but one can already see some nice features. One is that the only information about $f$ that enters into the new action is the singularity structure, i.e. the number and locations of zeros and poles. All functions $f$ with the same singularities should describe the same vacuum of the theory. The new formulation we propose makes this fact manifest.



JHEP08(2003)017

This transformation is similar in some ways to the Seiberg-Witten map of non-commutative gauge theory [17]. That is the equivalence of two theories, one with some background field turned on (effecting the kinetic term) with another theory with no background field, but a different multiplication rule. Here too we can formulate the theory with either a new kinetic operator, or a new multiplication rule.

For a certain function $f(\sigma) = \sin^2 \sigma$, there is no cohomology at ghost number one. Therefore a double zero at the boundary should correspond to the closed string vacuum. We conjecture that a different number (or order) of zeros of $f$ will correspond to different vacua of the theory, and in particular other functions will correspond to vacua with more D-branes.

If we relate the order of the zeros of $f$ to the number of D-brane, it is tempting to think that there should be a way to calculate the action of the brane from it. Perhaps using the techniques similar to those of section 5 one could express the action for the classical solution in terms of an index of the function $f$. So far we have been unable to do that.


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